

# Hawking radiation in a Rotating Kaluza-Klein Black Hole with Squashed Horizons

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## Abstract

We explore the signature of the extra dimension in the Hawking radiation in a rotating Kaluza-Klein black hole with squashed horizons. Comparing with the spherical case, we find that the rotating parameter brings richer physics. We obtain the appropriate size of the extra dimension which can enhance the Hawking radiation and may open a window to detect the extra dimensions.

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## I. INTRODUCTION

String theory is believed as the promising candidate for the unified theory of everything, which predicts the existence of the extra dimension. It is of great interest to study whether the extra dimension can be observed. This can present the signature of string and the correctness of string theory.

A great deal of effort has been expended for the detection of the extra dimension. One among them is the study of the perturbation around braneworld black holes. It has been argued that the extra dimension can imprint in the wave dynamics in the braneworld black hole background [1, 2, 3, 4]. Another chief possibility of observing the extra dimension is the spectrum of Hawking radiation which is expected to be detected in particle accelerator experiments [5, 6, 7, 8, 9, 10, 11, 12, 13]. Recently through the study of Hawking radiation from squashed Kaluza-Klein (KK) black holes [14], it was argued that the luminosity of Hawking radiation can tell us the size of the extra dimension which opens a window to observe extra dimensions [15].

Besides the charged static KK black hole with squashed horizon found in five-dimensional Einstein-Maxwell theory [14], other solutions of squashed KK black hole [16, 17, 18] have also been obtained subsequently. A nice review on the KK black hole including its stability, phase diagram and thermodynamics can be found in [19] and references therein. Recently, an extension has been made to the rotating case with two equal angular momenta in Einstein theory with zero cosmological constant[20]. The spacetime of the rotating KK black hole solution with squashed horizons is geodesic complete and free of naked singularities. It has the similar topology and asymptotical structure to that of the static squashed KK black hole, but with richer physical properties. In this paper we are going to study the Hawking radiation in the rotating squashed KK black hole background. We will calculate the greybody factor of scalar particles propagating in this rotating squashed KK black hole and show the rich physics brought by the rotating parameter  $a$ .

## II. MASTER EQUATION IN THE SQUASHED KERR BLACK HOLE

The five-dimensional rotating squashed KK black hole with two equal angular momenta is described by[20]

$$ds^2 = -dt^2 + \frac{\Sigma_0}{\Delta_0} k(r)^2 dr^2 + \frac{r^2 + a^2}{4} [k(r)(\sigma_1^2 + \sigma_2^2) + \sigma_3^2] + \frac{M}{r^2 + a^2} (dt - \frac{a}{2} \sigma_3)^2, \quad (1)$$

with

$$\begin{aligned}
\sigma_1 &= -\sin\psi d\theta + \cos\psi \sin\theta d\phi, \\
\sigma_2 &= \cos\psi d\theta + \sin\psi \sin\theta d\phi, \\
\sigma_3 &= d\psi + \cos\theta d\phi,
\end{aligned} \tag{2}$$

where  $0 < \theta < \pi$ ,  $0 < \phi < 2\pi$  and  $0 < \psi < 4\pi$ . The parameters are given by

$$\begin{aligned}
\Sigma_0 &= r^2(r^2 + a^2), \\
\Delta_0 &= (r^2 + a^2)^2 - Mr^2, \\
k(r) &= \frac{(r_\infty^2 - r_+^2)(r_\infty^2 - r_-^2)}{(r_\infty^2 - r^2)^2}.
\end{aligned} \tag{3}$$

Here  $M$  and  $a$  correspond to mass and angular momenta, respectively.  $r = r_+$  and  $r = r_-$  are outer and inner horizons of the black hole and they relate to  $M, a$  by  $a^4 = (r_+ r_-)^2$ ,  $M - 2a^2 = r_+^2 + r_-^2$ .  $r_\infty$  corresponds to the spatial infinity. In the parameter space  $0 < r_- \leq r_+ < r_\infty$ ,  $r$  is restricted within the range  $0 < r < r_\infty$ . The shape of black hole horizon is deformed by the parameter  $k(r_+)$ .

In the metric (1), the intrinsic singularity is just the one at  $r = 0$ , while  $r_\pm$  and  $r_\infty$  are coordinate singularities. This can be seen by introducing a new radial coordinate  $\rho$  as [20]

$$\rho = \rho_0 \frac{r^2}{r_\infty^2 - r^2}, \tag{4}$$

with

$$\begin{aligned}
\rho_0^2 &= \frac{k_0}{4}(r_\infty^2 + a^2), \\
k_0 &= k(r = 0) = \frac{(r_\infty^2 + a^2)^2 - Mr_\infty^2}{r_\infty^4}.
\end{aligned} \tag{5}$$

At the spatial infinity  $r \rightarrow r_\infty, \rho \rightarrow \infty$ . Thus in the new coordinate,  $\rho$  varies from 0 to  $\infty$  when  $r$  changes from 0 to  $r_\infty$ . The metric (1) can be rewritten as

$$ds^2 = -\frac{(r_\infty^2 + a^2)^4}{4\rho_0^2 r_\infty^6} d\tau^2 + U d\rho^2 + R^2(\sigma_1^2 + \sigma_2^2) + W^2 \sigma_3^2 + V \left[ \frac{(r_\infty^2 + a^2)^2}{2\rho_0 r_\infty^3} d\tau - \frac{a}{2} \sigma_3 \right]^2, \tag{6}$$

where we have defined the proper time  $\tau = 2\rho_0 r_\infty^3 / (r_\infty^2 + a^2)^2 t$  for the observer at infinity. The quantities  $K$ ,

$V$ ,  $W$ ,  $R$  and  $U$  are functions of  $\rho$

$$\begin{aligned}
K^2 &= \frac{\rho + \rho_0}{\rho + \frac{a^2}{r_\infty^2 + a^2} \rho_0}, \\
V &= \frac{M}{r_\infty^2 + a^2} K^2, \\
W^2 &= \frac{r_\infty^2 + a^2}{4K^2} = \frac{M}{4V}, \\
R^2 &= \frac{(\rho + \rho_0)^2}{K^2}, \\
U &= \left( \frac{r_\infty^2}{r_\infty^2 + a^2} \right)^2 \times \frac{\rho_0^2}{W^2 - \frac{r_\infty^2}{4} \frac{\rho}{\rho + \rho_0} V}.
\end{aligned} \tag{7}$$

The Hawking temperature for this rotating squashed KK black hole is given by

$$T_H = \frac{(r_+^2 - r_-^2)r_+}{2\pi(r_+^2 + a^2)^2} \sqrt{1 - \frac{r_+^2}{r_\infty^2} \left[ \frac{r_\infty^2 + a^2}{r_\infty^2 - r_-^2} \right]^{3/2}} = \frac{\rho_+ - \rho_-}{4\pi(\rho_+ + \frac{a^2}{r_\infty^2 + a^2} \rho_0)^2} \sqrt{\frac{\rho_+}{\rho_+ + \rho_0}}, \tag{8}$$

where  $\rho_\pm = \rho_0 \frac{r_\pm^2}{r_\infty^2 - r_\pm^2}$  are the outer and inner horizons of the black hole in the new coordinate. The angular velocity at the event horizon is

$$\Omega_H = \frac{aK(\rho_+)^2(r_\infty^2 + a^2)}{\rho_0 r_\infty^3}, \tag{9}$$

The determinant of the metric is

$$g = -\frac{(r_\infty^2 + a^2)^2(\rho + \rho_0)^4}{4r_\infty^2 K^4} \sin^2 \theta, \tag{10}$$

and non-zero metric coefficients read

$$\begin{aligned}
g^{00} &= -\frac{4\rho_0^2 r_\infty^6}{(r_\infty^2 + a^2)^4} \left[ \frac{\Delta + M(r_\infty^2 + a^2)^2(\rho + \rho_0)^2 / (4\rho_0^2 r_\infty^4 K^2)}{\Delta} \right], \\
g^{11} &= \frac{K^2 \Delta}{(\rho + \rho_0)^2}, & g^{22} &= \frac{K^2}{(\rho + \rho_0)^2}, \\
g^{33} &= \frac{K^2}{(\rho + \rho_0)^2 \sin^2 \theta}, & g^{34} &= -\frac{K^2 \cos \theta}{(\rho + \rho_0)^2 \sin^2 \theta}, \\
g^{04} &= -\frac{Ma(\rho + \rho_0)^2}{\rho_0 r_\infty (r_\infty^2 + a^2) \Delta}, \\
g^{44} &= \frac{K^2 \cos^2 \theta}{(\rho + \rho_0)^2 \sin^2 \theta} + \frac{(r_\infty^2 + a^2)(\rho + \rho_0)[(r_\infty^2 + a^2)\rho + a^2 \rho_0 - M(\rho + \rho_0)]}{\rho_0^2 r_\infty^4 \Delta},
\end{aligned} \tag{11}$$

where

$$\Delta = \frac{[(r_\infty^2 + a^2)\rho + a^2 \rho_0]^2 - Mr_\infty^2 \rho(\rho + \rho_0)}{(r_\infty^2 + a^2)^2 - Mr_\infty^2}. \tag{12}$$

We note that  $g^{03}$  vanishes, which is different from that in the usual five dimensional Kerr black hole with different angular momenta. This implies that there exists some special properties in such spacetime.

The wave equation for the massless scalar field  $\Phi(t, r, \theta, \phi, \psi)$  in the background (6) obeys

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu) \Phi(t, r, \theta, \phi, \psi) = 0. \quad (13)$$

Taking the ansatz  $\Phi(t, r, \theta, \phi, \psi) = e^{-i\omega t} R(\rho) e^{im\phi + i\lambda\psi} S(\theta)$ , where  $S(\theta)$  is the so-called spheroidal harmonics, we can obtain the equation

$$\frac{1}{\sin\theta} \frac{d}{d\theta} \left[ \sin\theta \frac{d}{d\theta} \right] S(\theta) - \left[ \frac{(m - \lambda \cos\theta)^2}{\sin^2\theta} - E_{lm\lambda} \right] S(\theta) = 0, \quad (14)$$

for angular part. Obviously, this angular equation is independent of the rotating parameter  $a$ , and is exactly identical to that in the static squashed KK black hole spacetime [15]. This is not surprising and in [21] the same angular equation as that in the Schwarzschild spacetime was also obtained in the five-dimensional Kerr black hole with two equal rotational parameters. The eigenvalue of the angular equation (14) is  $E_{lm\lambda} = l(l+1) - \lambda^2$ .

The radial part reads

$$\frac{d}{d\rho} \left[ \Delta \frac{dR(\rho)}{d\rho} \right] + \left[ \frac{\tilde{K}^2}{\Delta} + \Lambda - E_{lm\lambda} \right] R(\rho) = 0, \quad (15)$$

with

$$\tilde{K}^2 = \frac{Mr_\infty^2(\rho + \rho_0)^4}{K^4(r_\infty^2 + a^2)^2} \left[ \omega - \frac{\lambda a K^2(r_\infty^2 + a^2)}{\rho_0 r_\infty^3} \right]^2, \quad (16)$$

$$\Lambda = \frac{4\rho_0^2 r_\infty^6 (\rho + \rho_0)^2}{K^2(r_\infty^2 + a^2)^4} \omega^2 - \frac{4\lambda^2 (\rho + \rho_0)^2}{r_\infty^2 + a^2}. \quad (17)$$

By solving equation (15), one can obtain the radial part of the wave-function  $R(\rho)$ , and then compute the absorption probability  $|\mathcal{A}_{lm\lambda}|^2$  and study the Hawking radiation of a scalar particle propagating in the black hole spacetime.

### III. GREYBODY FACTOR IN THE LOW-ENERGY REGIME

In this section, we will obtain the solution to the field equation by employing matching techniques on expressions valid in the near horizon ( $\rho \sim \rho_+$ ) and far field ( $\rho \gg \rho_+$ ) regimes in the low energy and low angular momentum limit.

Let us first focus on the near-horizon regime. In order to express equation (15) into the form of a known differential equation, we make the following change of variable

$$\rho \rightarrow f = \frac{\Delta K^4}{(\rho + \rho_0)^2} \Rightarrow \frac{df}{d\rho} = (1 - f) \frac{A K^2}{(\rho + \rho_0)}, \quad (18)$$

where

$$A = 1 - \frac{\rho_0 r_\infty^2 a^2}{\rho(r_\infty^4 - a^4) - a^4 \rho_0}. \quad (19)$$

The equation (15) near the horizon ( $\rho \sim \rho_+$ ) can be expressed as

$$f(1-f)\frac{d^2 R(f)}{df^2} + (1-D_*f)\frac{dR(f)}{df} + \left\{ \frac{K_*^2}{A(\rho_+)^2(1-f)f} - \frac{E_{lm\lambda} - \Lambda(\rho_+)}{A(\rho_+)^2(1-f)} \right\} R(f) = 0, \quad (20)$$

where

$$\begin{aligned} K_* &= \sqrt{1 + \frac{\rho_0}{\rho_+}} \left[ \rho_+ + \frac{a^2}{r_\infty^2 + a^2} \rho_0 \right] \left[ \omega - \frac{a\lambda K(\rho_+)^2 (r_\infty^2 + a^2)}{\rho_0 r_\infty^3} \right], \\ D_* &= 2 - \frac{1}{A(\rho_+)} - \frac{(\rho_+ + \rho_0)A'(\rho_+)}{K(\rho_+)^2 A(\rho_+)^2}. \end{aligned} \quad (21)$$

Here  $A'(\rho_+)$  denotes the derivative of  $A$  with respect to  $\rho$  at the outer horizon  $\rho = \rho_+$ . Redefining the field  $R(f) = f^\alpha(1-f)^\beta F(f)$ , we can write equation (20) into the form of hypergeometric equation

$$f(1-f)\frac{d^2 F(f)}{df^2} + [c - (1+a_1+b)f]\frac{dF(f)}{df} - a_1bF(f) = 0, \quad (22)$$

with

$$a_1 = \alpha + \beta + D_* - 1, \quad b = \alpha + \beta, \quad c = 1 + 2\alpha. \quad (23)$$

Due to the constraint from coefficient of  $F(f)$ , the power coefficients  $\alpha$  and  $\beta$  must satisfy the second-order algebraic equations

$$\alpha^2 + \frac{K_*^2}{A(\rho_+)^2} = 0, \quad (24)$$

and

$$\beta^2 + \beta(D_* - 2) + \frac{K_*^2}{A(\rho_+)^2} - \frac{E_{lm\lambda} - \Lambda(\rho_+)}{A(\rho_+)^2} = 0, \quad (25)$$

respectively. Solving these two equations, we obtain the solutions for the parameter  $\alpha$  and  $\beta$

$$\alpha_\pm = \pm \frac{iK_*}{A(\rho_+)}, \quad (26)$$

$$\beta_\pm = \frac{1}{2} \left[ (2 - D_*) \pm \sqrt{(D_* - 2)^2 - \frac{4K_*^2}{A(\rho_+)^2} + \frac{4(E_{lm\lambda} - \Lambda(\rho_+))}{A(\rho_+)^2}} \right]. \quad (27)$$

The general solution of the master equation (15) near the horizon can be expressed as

$$R_{NH}(f) = A_- f^\alpha (1-f)^\beta F(a_1, b, c; f) + A_+ f^{-\alpha} (1-f)^\beta F(a_1 - c + 1, b - c + 1, 2 - c; f), \quad (28)$$

where  $A_\pm$  are arbitrary constants. Near the horizon,  $\rho \rightarrow \rho_+$ , and  $f \rightarrow 0$ , the solution (28) can be reduced to

$$R_{NH}(f) = A_- f^{\pm iK_*/A(\rho_+)} + A_+ f^{\mp iK_*/A(\rho_+)} = A_- e^{\pm iK y} + A_+ e^{\mp iK y}, \quad (29)$$

with

$$\mathcal{K} = \sqrt{1 + \frac{\rho_0}{\rho_+}} \left[ \rho_+ + \frac{a^2}{r_\infty^2 + a^2} \rho_0 \right]^2 \left[ \omega - \frac{a\lambda K(\rho_+)^2 (r_\infty^2 + a^2)}{\rho_0 r_\infty^3} \right] \quad (30)$$

where  $y$  is the tortoise-like coordinate and can be expressed as

$$y = \frac{K(\rho_+)^2 \ln f}{A(\rho_+)(\rho_+ + \rho_0)}. \quad (31)$$

In the limit  $\rho \rightarrow \rho_+$ , it becomes identical to the tortoise coordinate  $\rho_*$  defined by  $d\rho_*/d\rho = K^4/f(\rho + \rho_0)^2$  as in [15]. Thus, the factors  $f^{\pm iK_*/A(\rho_+)}$  in the near-horizon asymptotic solution can be reduced to  $e^{\pm i\mathcal{K}y}$ , which describes an outgoing and incoming free waves, respectively. In terms of the boundary condition that no outgoing mode exists near the horizon, we choose  $\alpha = \alpha_-$  and  $A_+ = 0$ , which results in the asymptotic solution near horizon

$$R_{NH}(f) = A_- f^\alpha (1-f)^\beta F(a_1, b, c; f). \quad (32)$$

Moreover, the above boundary condition also demands that near the horizon the hypergeometric function  $F(a_1, b, c; f)$  must be convergent, i.e.  $Re(c - a_1 - b) > 0$ , which implies that we must choose  $\beta = \beta_-$ .

In order to match the near horizon and far field solutions in the intermediate zone, we must stretch the near horizon solution to the large value of the radial coordinate. As done in Ref.[23], at first we change the argument of the hypergeometric function of the near-horizon solution from  $f$  to  $1-f$  by using the relation

$$\begin{aligned} R_{NH}(f) = & A_- f^\alpha (1-f)^\beta \left[ \frac{\Gamma(c)\Gamma(c-a_1-b)}{\Gamma(c-a_1)\Gamma(c-b)} F(a_1, b, a_1+b-c+1; 1-f) \right. \\ & \left. + (1-f)^{c-a_1-b} \frac{\Gamma(c)\Gamma(a_1+b-c)}{\Gamma(a_1)\Gamma(b)} F(c-a_1, c-b, c-a_1-b+1; 1-f) \right]. \end{aligned} \quad (33)$$

In the limit  $\rho \gg \rho_+$ , the function  $(1-f)$  can be approximated by

$$1-f \simeq \frac{M(r_\infty^2 - a^2)}{4\rho_0 r_\infty^2 \rho}, \quad (34)$$

and the near-horizon solution (33) can be simplified further to

$$R_{NH}(\rho) \simeq A_1 \rho^{-\beta} + A_2 \rho^{\beta+D_*-2}, \quad (35)$$

with

$$A_1 = A_- \left[ \frac{M(r_\infty^2 - a^2)}{4\rho_0 r_\infty^2} \right]^\beta \frac{\Gamma(c)\Gamma(c-a_1-b)}{\Gamma(c-a_1)\Gamma(c-b)}, \quad (36)$$

$$A_2 = A_- \left[ \frac{M(r_\infty^2 - a^2)}{4\rho_0 r_\infty^2} \right]^{-(\beta+D_*-2)} \frac{\Gamma(c)\Gamma(a_1+b-c)}{\Gamma(a_1)\Gamma(b)}. \quad (37)$$

Now, let us turn to the far field region ( $\rho \rightarrow \infty$ ), where the equation (15) reduces to

$$\frac{d^2 R_{FF}(\rho)}{d\rho^2} + \frac{2}{\rho} \frac{dR_{FF}(\rho)}{d\rho} + \left[ \tilde{\omega}^2 - \frac{E_{lm\lambda}}{\rho^2} \right] R_{FF}(\rho) = 0, \quad (38)$$

with

$$\tilde{\omega}^2 = \frac{4\rho_0^2 r_\infty^6 \omega^2}{(r_\infty^2 + a^2)^4} - \frac{4\lambda^2}{r_\infty^2 + a^2} + \frac{Mr_\infty^2}{(r_\infty^2 + a^2)^2} \left[ \omega - \frac{a\lambda(r_\infty^2 + a^2)}{\rho_0 r_\infty^3} \right]^2. \quad (39)$$

Obviously, it is a Bessel equation. Thus, the general solution of radial master equation (15) in the far field region can be expressed as

$$R_{FF}(\rho) = \frac{1}{\sqrt{\rho}} \left[ B_1 J_\nu(\tilde{\omega}\rho) + B_2 Y_\nu(\tilde{\omega}\rho) \right], \quad (40)$$

where  $J_\nu(\tilde{\omega}\rho)$  and  $Y_\nu(\tilde{\omega}\rho)$  are the first and second kind Bessel functions,  $\nu = \sqrt{E_{lm\lambda} + 1/4}$ .  $B_1$  and  $B_2$  are integration constants. In the limit  $\rho \rightarrow 0$ ,  $R_{FF}(\rho)$  in equation (40) becomes

$$R_{FF}(\rho) \simeq \frac{B_1 (\frac{\tilde{\omega}\rho}{2})^\nu}{\sqrt{\rho} \Gamma(\nu + 1)} - \frac{B_2 \Gamma(\nu)}{\pi \sqrt{\rho} (\frac{\tilde{\omega}\rho}{2})^\nu}. \quad (41)$$

Comparing it with equation (35), we can obtain two relations between  $A_1$  and  $B_1$ ,  $B_2$  in the limit  $\omega\rho_+ \ll 1$ .

Employing equations (36) and (37) and removing  $A_-$ , we find the constraint for  $B_1$ ,  $B_2$

$$\begin{aligned} B \equiv \frac{B_1}{B_2} &= -\frac{1}{\pi} \left[ \frac{8\rho_0 r_\infty^2}{M\tilde{\omega}(r_\infty^2 - a^2)} \right]^{\sqrt{4E_{lm\lambda} + 1}} \sqrt{E_{lm\lambda} + 1/4} \\ &\times \frac{\Gamma^2(\sqrt{E_{lm\lambda} + 1/4}) \Gamma(c - a_1 - b) \Gamma(a_1) \Gamma(b)}{\Gamma(a_1 + b - c) \Gamma(c - a_1) \Gamma(c - b)}. \end{aligned} \quad (42)$$

In the asymptotic region  $\rho \rightarrow \infty$ , the solution in the far field can be expressed as

$$R_{FF}(\rho) \simeq \frac{B_1 + iB_2}{2\sqrt{2\pi\tilde{\omega}\rho}} e^{-i\tilde{\omega}\rho} + \frac{B_1 - iB_2}{2\sqrt{2\pi\tilde{\omega}\rho}} e^{i\tilde{\omega}\rho} = A_{in}^{(\infty)} \frac{e^{-i\tilde{\omega}\rho}}{\rho} + A_{out}^{(\infty)} \frac{e^{i\tilde{\omega}\rho}}{\rho}. \quad (43)$$

We need to point out that only when the condition  $\tilde{\omega} > 0$  is satisfied, the solution (43) denotes an incoming and an outgoing spherical waves for large distances from the black hole.

The absorption probability can be calculated by

$$|\mathcal{A}_{lm\lambda}|^2 = 1 - \left| \frac{A_{out}^{(\infty)}}{A_{in}^{(\infty)}} \right|^2 = 1 - \left| \frac{B - i}{B + i} \right|^2 = \frac{2i(B^* - B)}{BB^* + i(B^* - B) + 1}. \quad (44)$$

Combining the above result and the expression  $B$  given in equation (42), we can analyse the properties of absorption probability for the massless scalar field in a rotating squashed KK black hole background in the low-energy and low-angular momentum limits.

In Fig.(1), we plot the absorption probability for the first partial waves ( $l = 0, \lambda = 0$ ) by fixing  $a = 0.2$ . One can easily see that the absorption probability decreases with the increase of the parameter  $r_\infty$ , which is similar



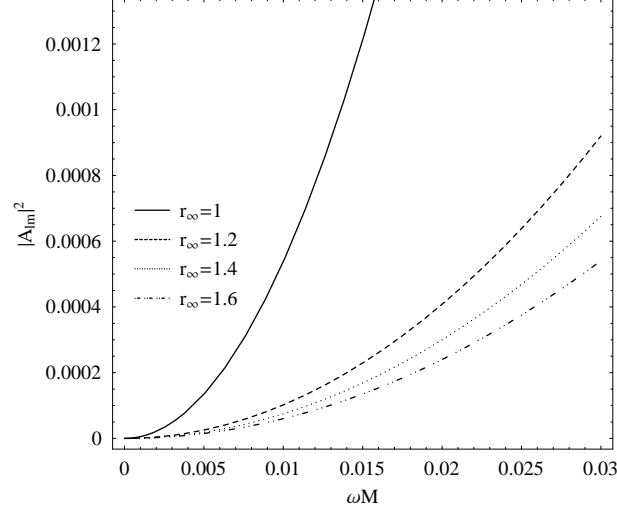


FIG. 1: Absorption probability  $|\mathcal{A}_{lm\lambda}|^2$  of scalar particles propagating in the rotating squashed Kaluza-Klein black hole spacetime, for fixed  $l = 0$ ,  $\lambda = 0$ ,  $a = 0.2$  and different  $r_\infty$ .

to that in the nonrotating case shown in [15] where Eq.(31) can be written as  $|\mathcal{A}|^2 = \frac{\omega^2 r_\infty r_+^3}{r_\infty^2 - r_+^2}$  decreasing with the increase of  $r_\infty$ . The main reason behind this phenomenon is that the larger  $r_\infty$  yields the lower peak of the effective potential so that more radiation can be transmitted to the infinity.

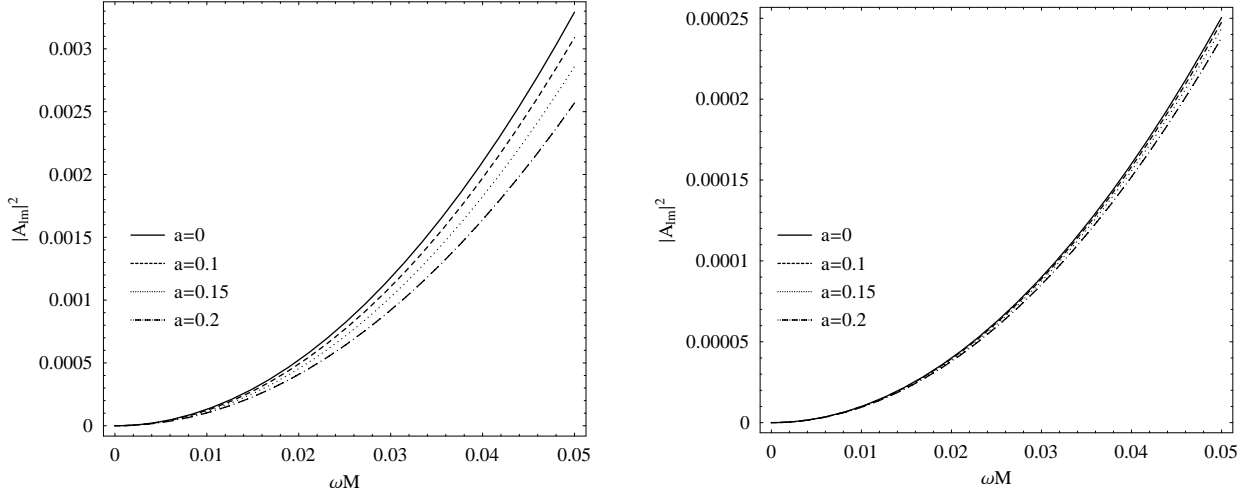


FIG. 2: Absorption probability  $|\mathcal{A}_{lm\lambda}|^2$  (the left  $r_\infty = 1.2$  and the right  $r_\infty = 5$ ) of scalar particles propagating in the rotating squashed Kaluza-Klein black hole spacetime, for fixed  $l = 0$ ,  $\lambda = 0$  and different  $a$ .

In figure (2) we show the dependence of the absorption probability on the angular momentum parameter. We see that for fixed  $r_\infty$ , the absorption probability decreases with the increase of the angular momentum parameter  $a$ . The suppression of the absorption due to the increase of  $a$  is very similar to that of the Reissner-Nordstrom

black hole when the value of  $q$  increases [22]. For the fixed  $r_\infty$ , we observed that for larger  $a$ , the potential peak becomes lower, which allows more radiation to leak to the infinity. The difference of the absorption probability due to different  $a$  becomes smaller when the value of  $r_\infty$  increases. This is because that big  $r_\infty$  causes the spectrum to be suppressed as observed in Fig.1.

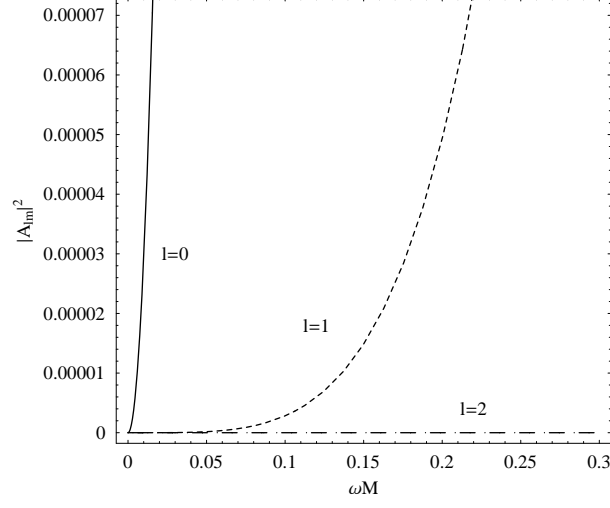


FIG. 3: Absorption probability  $|\mathcal{A}_{lm\lambda}|^2$  of scalar particles propagating in the rotating squashed Kaluza-Klein black hole spacetime, for fixed  $r_\infty = 2$ ,  $\lambda = 0$ ,  $a = 0.2$  and different  $l$ .

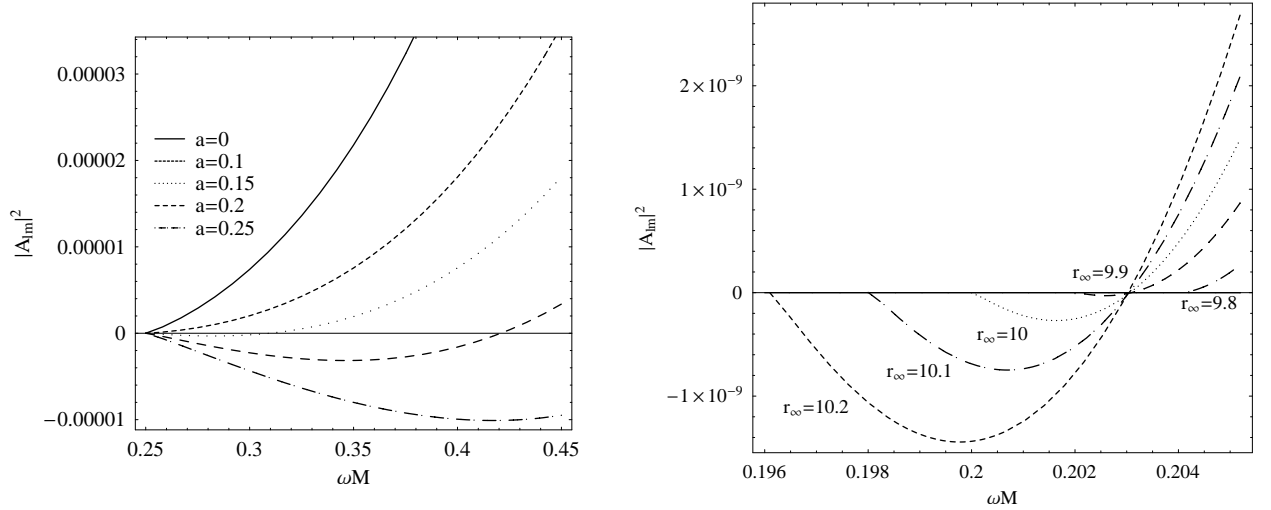


FIG. 4: Absorption probability  $|\mathcal{A}_{lm\lambda}|^2$  of scalar particles propagating in the rotating squashed Kaluza-Klein black hole spacetime, for  $l = 1$ ,  $\lambda = 1$ . The left is for fixed  $r_\infty = 5$  and different  $a$ , and the right is for fixed  $a = 0.1$  and different  $r_\infty$ .

Fig.3 shows the dependence of the absorption probability on the angular index  $l$ . We see the dominance of the low angular index spectrum over others. This property has also been observed in [23]. In the low energy

approximation, (47) in [23] leads the absorption probability  $|\mathcal{A}|^2 \sim \omega^{2l+2}$ , which tells us that in the limit  $\omega \rightarrow 0$ , the suppression of  $|\mathcal{A}|^2$  as the value of the angular index increases. The dominance of the absorption probability at the low angular index numbers has also been shown numerically in [24].

In Fig.4, we find that for positive  $\lambda$ , in some range of frequencies  $\omega$ , the absorption probability can be negative, which presents us the super-radiance. This property has not been observed in the spherical squashed KK black hole in [15].

As in [23], in the low energy limit  $BB^* \gg i(B^* - B) \gg 1$ , we can simplify our (44) to the form

$$|\mathcal{A}_{lm\lambda}|^2 = 2i\left(\frac{1}{B} - \frac{1}{B^*}\right) = 4\pi \left[ \frac{M\tilde{\omega}(r_\infty^2 - a^2)}{8\rho_0 r_\infty^2} \right]^{\sqrt{4E_{lm\lambda} + 1}} \frac{K_*}{A(\rho_+)} \frac{\Gamma^2(2\beta + D_* - 2)\Gamma^2(1 - \beta)(2 - D_* - 2\beta)}{\sqrt{E_{lm\lambda} + 1/4}\Gamma^2(\sqrt{E_{lm\lambda} + 1/4})\Gamma^2(\beta + D_* - 1)\sin^2(\pi(\beta + D_*))}. \quad (45)$$

From (27) we learnt that the quantity  $2 - D_* - 2\beta$  is always positive. Using (19) we have  $A(\rho_+) = 1 - \frac{\rho_0 r_\infty^2 a^2}{\rho_+ (r_\infty^4 - a^4) - a^4 \rho_0} = \frac{(r_\infty^2 + a^2)(r_+^2 - a^2)}{r_+^2 (r_\infty^2 - r_-^2)}$ , which is positive since  $r_+^2 - a^2 = (M - 4a^2 + \sqrt{M(M - 4a^2)})/2 > 0$ .  $\tilde{\omega} > 0$  is required to describe the outgoing and incoming spherical waves in the large distance in (43). The possibility to make  $|\mathcal{A}_{lm\lambda}|^2 < 0$  is  $K_* < 0$ . From (21) and (39),  $K_* < 0$  and  $\tilde{\omega} > 0$  lead to

$$0 \leq \omega \leq \omega_c = \frac{a\lambda K(\rho_+)^2(r_\infty^2 + a^2)}{\rho_0 r_\infty^3} = \frac{2a\lambda(r_\infty^2 + a^2)^2}{r_\infty(r_+^2 + a^2)\sqrt{(r_\infty^2 + a^2)[(r_\infty^2 + a^2)^2 - Mr_\infty^2]}}, \quad (46)$$

and

$$\omega \geq \omega_0 = \frac{2\lambda(r_\infty^2 + a^2)^2}{r_\infty[(r_\infty^2 + a^2)^2 + Ma^2]} \frac{\sqrt{(r_\infty^2 + a^2)[(r_\infty^2 + a^2)^2 - Mr_\infty^2]} + Ma}{\sqrt{(r_\infty^2 + a^2)[(r_\infty^2 + a^2)^2 - Mr_\infty^2]}}, \quad (47)$$

respectively. The condition for the occurrence of the super-radiance in this black hole background is  $\omega_0 \leq \omega_c$ .

From (46) and (47), we obtain the ratio

$$\frac{\omega_0}{\omega_c} = \frac{(r_+^2 + a^2)\{\sqrt{(r_\infty^2 + a^2)[(r_\infty^2 + a^2)^2 - Mr_\infty^2]} + Ma\}}{a[(r_\infty^2 + a^2)^2 + Ma^2]}. \quad (48)$$

The changes of the ratio with  $a$  and  $r_\infty$  are plotted in figure (5), which tell us that the larger value of the angular momentum parameter  $a$  leads to the more extended region of the super-radiance. This has also been observed in [23]. The increase of  $r_\infty$  gives the similar result. Moreover, we observed that for the fixed  $r_\infty$ , there exists a lower bound of  $a$  for the super-radiance to occur. Similarly, for the fixed  $a$ , there is also the lowest value of  $r_\infty$  for the super-radiance to happen. This is actually shown in Fig.4.

Now let us turn to study the luminosity of the Hawking radiation for the mode  $l = 0$ ,  $\lambda = 0$  which plays a dominant role in the greybody factor. Performing an analysis similar to that in [23], we can rewrite the

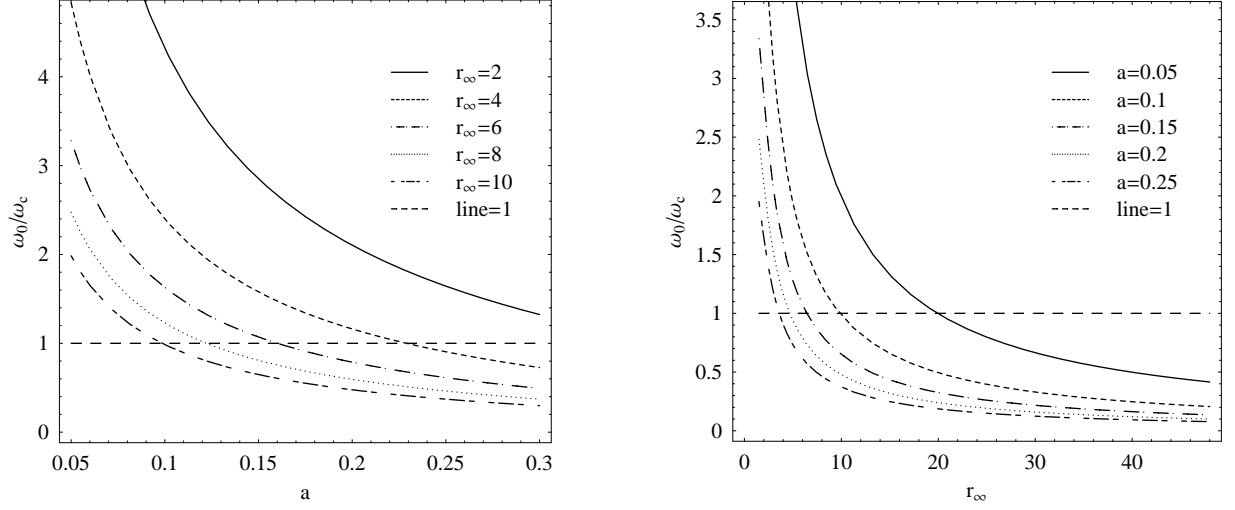


FIG. 5: Ratio of the value  $\omega_0/\omega_c$  with the change of  $a$  and  $r_\infty$ .

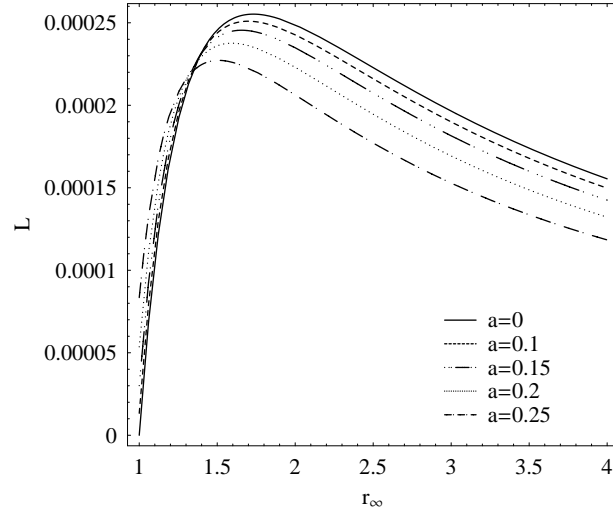


FIG. 6: The luminosity of Hawking radiation  $L$  of scalar particles propagating in the rotating squashed Kaluza-Klein black hole spacetime, for  $l = 0$  and  $\lambda = 0$  and different  $r_\infty$  and  $a$ .

greybody factor (44) as

$$\begin{aligned}
 |\mathcal{A}_{lm\lambda}|^2 &\simeq \frac{4\omega^2(\rho_0 + \rho_-)}{\rho_0\rho_+} \sqrt{1 + \frac{\rho_0}{\rho_+}} \left[ \rho_+ + \frac{a^2}{r_\infty^2 + a^2} \rho_0 \right]^3 \\
 &= \frac{\omega^2 r_\infty^5 (r_+^2 + a^2)^3}{r_+^3 (r_\infty^2 + a^2)^2 (r_\infty^2 - r_+^2)}.
 \end{aligned} \tag{49}$$

For the slowly rotating black hole, when  $r_\infty$  increases, the greybody factor decreases. In the limit  $a \rightarrow 0$ , we have  $\rho_- \rightarrow 0$ , and then the form of the formula (49) reduces to that in the static squashed Kaluza-Klein black

hole spacetime [15]. Combining it with equation (8), the luminosity of the Hawking radiation is given by

$$\begin{aligned}
 L &= \int_0^\infty \frac{d\omega}{2\pi} |\mathcal{A}_{lm\lambda}|^2 \frac{\omega}{e^{(\omega - \Omega_H \lambda)/T_H} - 1} \\
 &\simeq \frac{1}{1920\pi\rho_+^2} \left(1 + \frac{\rho_-}{\rho_0}\right) \left(1 - \frac{\rho_-}{\rho_+}\right)^4 \left(1 + \frac{\rho_0}{\rho_+}\right)^{-3/2} \left(1 + \frac{a^2}{r_\infty^2 + a^2} \frac{\rho_0}{\rho_+}\right)^{-5} \\
 &= \frac{r_+ r_\infty (r_\infty^2 - r_+^2) (r_\infty^2 + a^2)^4 (r_+^2 - r_-^2)^4}{480\pi (r_+^2 + a^2)^5 (r_\infty^2 - r_-^2)^6} = \frac{\pi^3}{30} G T_H^4,
 \end{aligned} \tag{50}$$

where  $G = \frac{r_\infty^5 (r_+^2 + a^2)^3}{r_+^3 (r_\infty^2 + a^2)^2 (r_\infty^2 - r_+^2)}$ . In figure 6, we show the dependence of the luminosity of Hawking radiation

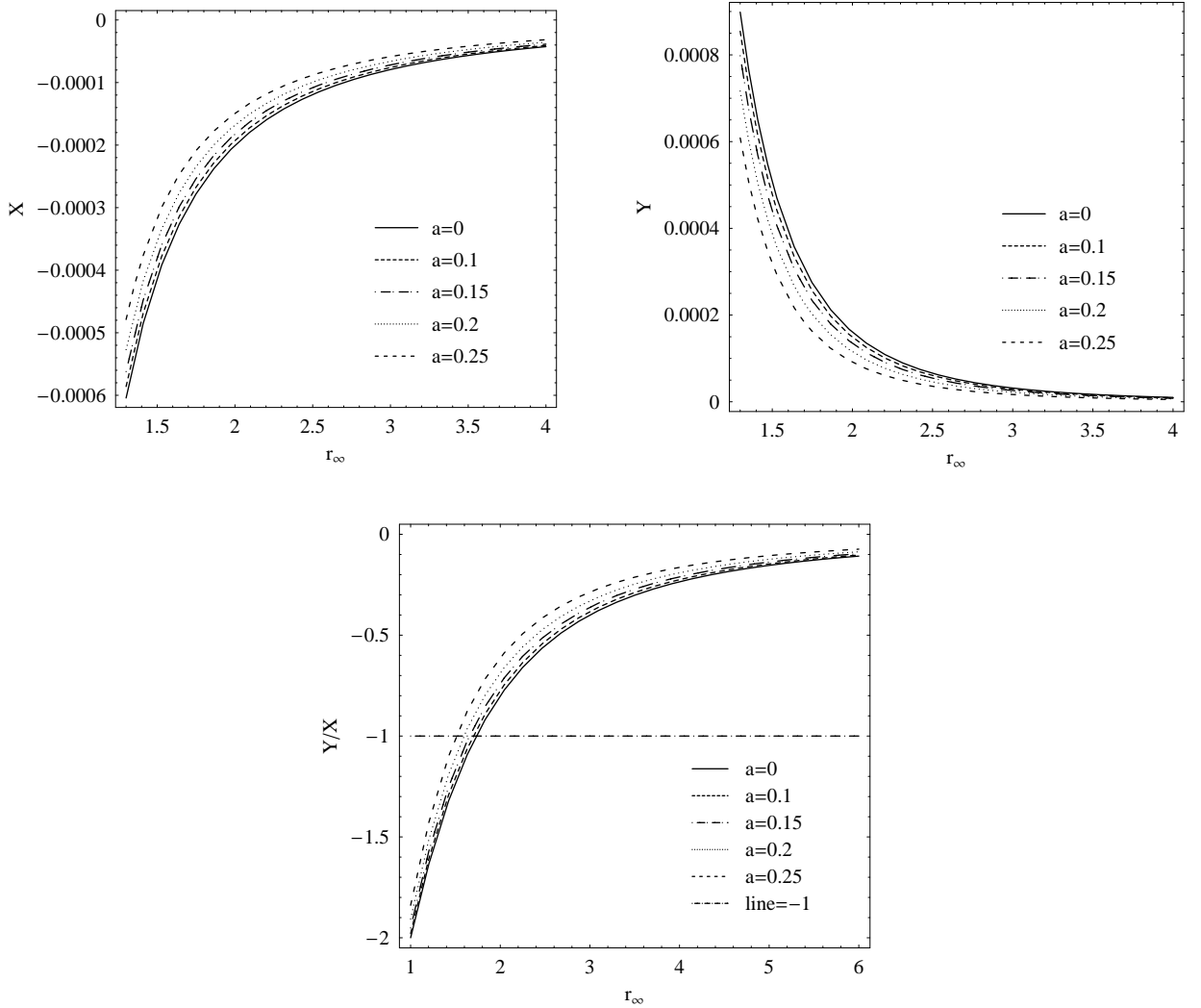


FIG. 7: The behaviors of  $X, Y$  and the ratio  $Y/X$  with the change of  $r_\infty$  and  $a$ .

on the size of the extra dimension  $r_\infty$  for different angular momentum parameters. In the limit  $r_+ \rightarrow r_\infty$ ,  $L \rightarrow 0$ . The limit  $r_+ \rightarrow r_\infty$  describes that  $r_+$  reaches the size of the fifth dimension at the infinity  $r_\infty$ , which means that we could effectively obtain a very large four-dimensional black hole. This limit tells us that the KK

$a$	0	0.1	0.15	0.2	0.25
$r_+$	1	0.9899	0.9770	0.9583	0.9330
$r_\infty^p$	1.7321	1.6989	1.6562	1.5938	1.5092
$\frac{(r_\infty^p)^2}{r_+^2}$	3	2.9454	2.8737	2.7664	2.6164

TABLE I: The change of  $r_+$ ,  $r_\infty^p$  and  $(r_\infty^p)^2/r_+^2$  with different  $a$ . Here  $M = 1$ .

black hole has an infinite horizon radius, as a result the temperature approaches zero, like the Schwarzschild black hole with infinite radius [25], which naturally leads to the vanishing of the luminosity of the Hawking radiation. When  $r_+ < r_\infty$ , from (50) we know that there is a peak of the luminosity of the Hawking radiation when  $dL/dr_\infty = 0$ . For fixed  $M$  but different  $a$ , peaks appear at different  $r_\infty^p$  are listed in table I. When  $r_+ < r_\infty < r_\infty^p$ , we observe in Fig.6 that with the increase of  $r_\infty$ ,  $L$  increases. This can be explained by the derivative  $dL/dr_\infty$  which can be expressed into  $\frac{\pi^3}{30}(X + Y)$ , where  $X = T_H^4 dG/dr_\infty$ ,  $Y = GdT_H^4/dr_\infty$ . The variations of  $X, Y$  and  $X/Y$  with the change of  $r_\infty$  are plotted in Fig.7.  $X$  is negative and  $Y$  is positive. For small values of  $r_\infty$  we have  $Y/X < -1$  (i.e  $X + Y > 0$ ), this leads  $dL/dr_\infty > 0$ . This shows that the extra dimensional effect is enhanced, which may allow us to detect the extra dimension. For the larger  $r_\infty$ , we have  $Y/X > -1$ , (i.e,  $X + Y < 0$ ) which leads  $dL/dr_\infty < 0$  and we see in Fig.6 that when  $r_\infty > r_\infty^p$  the luminosity of Hawking radiation decreases with the further increase of  $r_\infty$ .

#### IV. SUMMARY

In this paper, we employed the matching techniques and studied the low-energy greybody factor and Hawking radiation for a massless scalar field in the background of a 5-dimensional rotating squashed KK black hole. We found that the absorption probability and Hawking radiation contain the imprint of the extra dimensions. With the inclusion of the rotating parameter  $a$ , we have observed richer physics which has not been shown in the spherical squashed KK black hole background[15], such as the super-radiance and the dependence of the absorption probability and the luminosity of the Hawking radiation on the rotating parameter. We found that for an appropriate size of the extra dimension  $r_\infty$ , the signature of the extra dimension can be enhanced in the Hawking radiation. This could open a window to detect the extra dimension.

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